Random Stuff 1

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$0.1\dot{0}\dot{1} = 0.1010101\ldots$

$0.\dot{9} = x$

$\Rightarrow x = \frac{9}{9} = 1$

$\boxed{x = x' \text{ iff } x - x' < \epsilon \ \forall \epsilon > 0 \text{ if } x, x' \in \mathbb{R}}$

$0.a_1a_2\ldots a_m\dot{b}_1b_2\ldots\dot{b}_n = x$

$10^{m+n}x = a_1a_2\ldots a_mb_1\ldots b_n.\dot{b}_1\ldots\dot{b}_n$

$10^{m+n}x - 10^m x = (a_1a_2\ldots a_mb_1\ldots b_n - a_1\ldots a_m)$

$\Rightarrow (10^{m+n} - 10^m)x = (\ \ldots\ )$

$\Rightarrow x = \frac{(\ \ldots\ )}{10^{m+n} - 10^m}$

$= \frac{(\ \ldots\ )}{10^m \times \underbrace{9\ldots9}_{n}}$

Q) Prove that if $n > 4$ then $\underbrace{1! + 2! + 3! + \cdots + n!}_{S}$ is never a square

Ans- S is always odd for $n > 4$.

$S = (2k+1)^2 \Rightarrow S = 4k^2 + 4k + 1 = 4k(k+1) + 1 = 8m + 1$ (let)

$1! = 1$

$1! + 2! = 3$

$1! + 2! + 3! = 9$

$1! + 2! + 3! + 4! = 33$

$1! + 2! + 3! + 4! + 5! = 153$

for $n \geq 5$, $10 \mid (n!)$

$\Rightarrow S \equiv 33 \pmod{10}$

$\equiv 3 \pmod{10}$

Last digit

$0^2 = 0$, $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 6$, $5^2 = 5$, $6^2 = 6$, $7^2 = 9$, $8^2 = 4$, $9^2 = 1$,

$\Rightarrow S$ has 3 as last digit $\Rightarrow S$ is not a square

Q) Let d be any positive integer not equal to 2, 5, or 13. Show that we can find distinct a, b in the set $\{2, 5, 13, d\}$ such that $(ab - 1)$ is not a perfect square.

$\{2, 5, 13, d\}$ such that $(ab-1)$ is not a perfect square.

Square ends with 0, 1, 4, 5, 6, 9

Ans:– We have to use d

Let $2d-1$, $5d-1$, $13d-1$ be all perfect squares.

$= x^2 \quad = y^2 \quad = z^2$

$(2(k+1))^2 = 4k^2+4k+1$

Let d be even,

$x^2 \pmod 4 \equiv 1 \pmod 4$

But $d = 2m \Rightarrow 2(2m)-1 = 4m-1 \equiv 3 \pmod 4$ not possible

So if d is even then $a = 2$, $b = d$ will suffice

When d is odd,

$2d-1 = 2(2m+1)-1 = 4m+2-1 = 4m+1 \equiv 1 \pmod 4$

even $\Leftarrow$ $5d-1 = 5(2m+1)-1 = 10m+5-4 = 10m+1 \equiv 0 \pmod 4$

$\Rightarrow 4 \mid (5d-1)$ So d can be $4n+1$ or $4n+3$.

odd $\Leftarrow$ $2(4n+1)-1 = 8n+1 = x^2 \equiv 1 \pmod 4$

even $\Leftarrow$ $5(4n+1)-1 = 20n+4 = y^2 \equiv 0 \pmod 4$

even $\Leftarrow$ $13(4n+1)-1 = 52n+12 = z^2 \equiv 0 \pmod 4$

$\frac{1}{4}\, 4k^2 \Rightarrow k'^2$

$5n+1 = \left(\frac{y}{2}\right)^2 \equiv n+1 \pmod 4$

$13n+3 = \left(\frac{z}{2}\right)^2 \equiv n+3 \pmod 4$

$(2k+1)^2 = 1 \pmod 4$

$(2k)^2 = 0 \pmod 4$

If both $5n+1$ and $13n+3$ are perfect squares then,

If n is even then, $n+1$ or $n+3$ will be 1 or 3 $\pmod 4$ (not possible)

If n is odd then, $n+1$ or $n+3$ will be 0 or 2 $\pmod 4$ (not possible)

So contradiction.

Q) Find all primes p such that both p and p^2+8 are primes